Spring 2012

EE 330

ENGINEERING ELECTROMAGNETICS

HW 2: 1.28, and 2.1 through 2.8 **due 20 January**

Problem 1.28 A series RLC circuit is connected to a generator with a voltage $\upsilon_s(t) = V_0 \cos(\omega t + \pi/3)$ (V).

(a) Write the voltage loop equation in terms of the current $i(t)$, R, L, C, and $v_s(t)$.

(b) Obtain the corresponding phasor-domain equation.

(c) Solve the equation to obtain an expression for the phasor current $\widetilde{I}$.

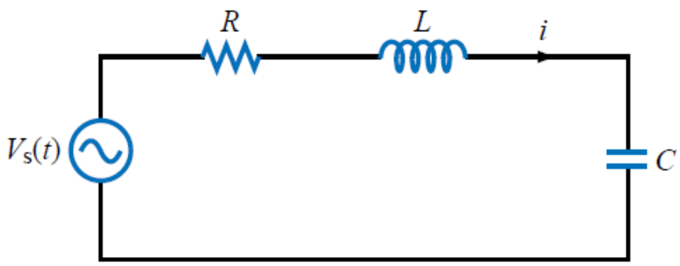


Figure P1.28: RLC circuit.

Solution:

(a) $\upsilon_s(t) = Ri + L\dfrac{di}{dt} + \dfrac{1}{C}\displaystyle\int i\,dt.$

(b) In phasor domain: $\widetilde{V}_s = R\tilde{I} + j\omega L\tilde{I} + \dfrac{\tilde{I}}{j\omega C}.$

(c) $$\tilde{I} = \frac{\widetilde{V}_s}{R + j(\omega L - 1/\omega C)} = \frac{V_0 e^{j\pi/3}}{R + j(\omega L - 1/\omega C)} = \frac{\omega C V_0 e^{j\pi/3}}{\omega RC + j(\omega^2 LC - 1)}.$$

Problem 2.1 A transmission line of length l connects a load to a sinusoidal voltage source with an oscillation frequency f. Assuming the velocity of wave propagation on the line is c, for which of the following situations is it reasonable to ignore the presence of the transmission line in the solution of the circuit:

(a) $l = 20$ cm, $f = 20$ kHz,
(b) $l = 50$ km, $f = 60$ Hz,
(c) $l = 20$ cm, $f = 600$ MHz,
(d) $l = 1$ mm, $f = 100$ GHz.

Solution: A transmission line is negligible when $l/\lambda \leq 0.01$.

(a) $$\frac{l}{\lambda} = \frac{lf}{u_{\text{p}}} = \frac{(20 \times 10^{-2} \text{ m}) \times (20 \times 10^{3} \text{ Hz})}{3 \times 10^{8} \text{ m/s}} = 1.33 \times 10^{-5} \text{ (negligible)}.$$

(b) $$\frac{l}{\lambda} = \frac{lf}{u_{\text{p}}} = \frac{(50 \times 10^{3} \text{ m}) \times (60 \times 10^{0} \text{ Hz})}{3 \times 10^{8} \text{ m/s}} = 0.01 \text{ (borderline)}.$$

(c) $$\frac{l}{\lambda} = \frac{lf}{u_{\text{p}}} = \frac{(20 \times 10^{-2} \text{ m}) \times (600 \times 10^{6} \text{ Hz})}{3 \times 10^{8} \text{ m/s}} = 0.40 \text{ (nonnegligible)}.$$

(d) $$\frac{l}{\lambda} = \frac{lf}{u_{\text{p}}} = \frac{(1 \times 10^{-3} \text{ m}) \times (100 \times 10^{9} \text{ Hz})}{3 \times 10^{8} \text{ m/s}} = 0.33 \text{ (nonnegligible)}.$$

Problem 2.2 A two-wire copper transmission line is embedded in a dielectric material with $\varepsilon_r = 2.6$ and $\sigma = 2 \times 10^{-6}$ S/m. Its wires are separated by 3 cm and their radii are 1 mm each.

(a) Calculate the line parameters R', L', G', and C' at 2 GHz.

(b) Compare your results with those based on CD Module 2.1. Include a printout of the screen display.

Solution:

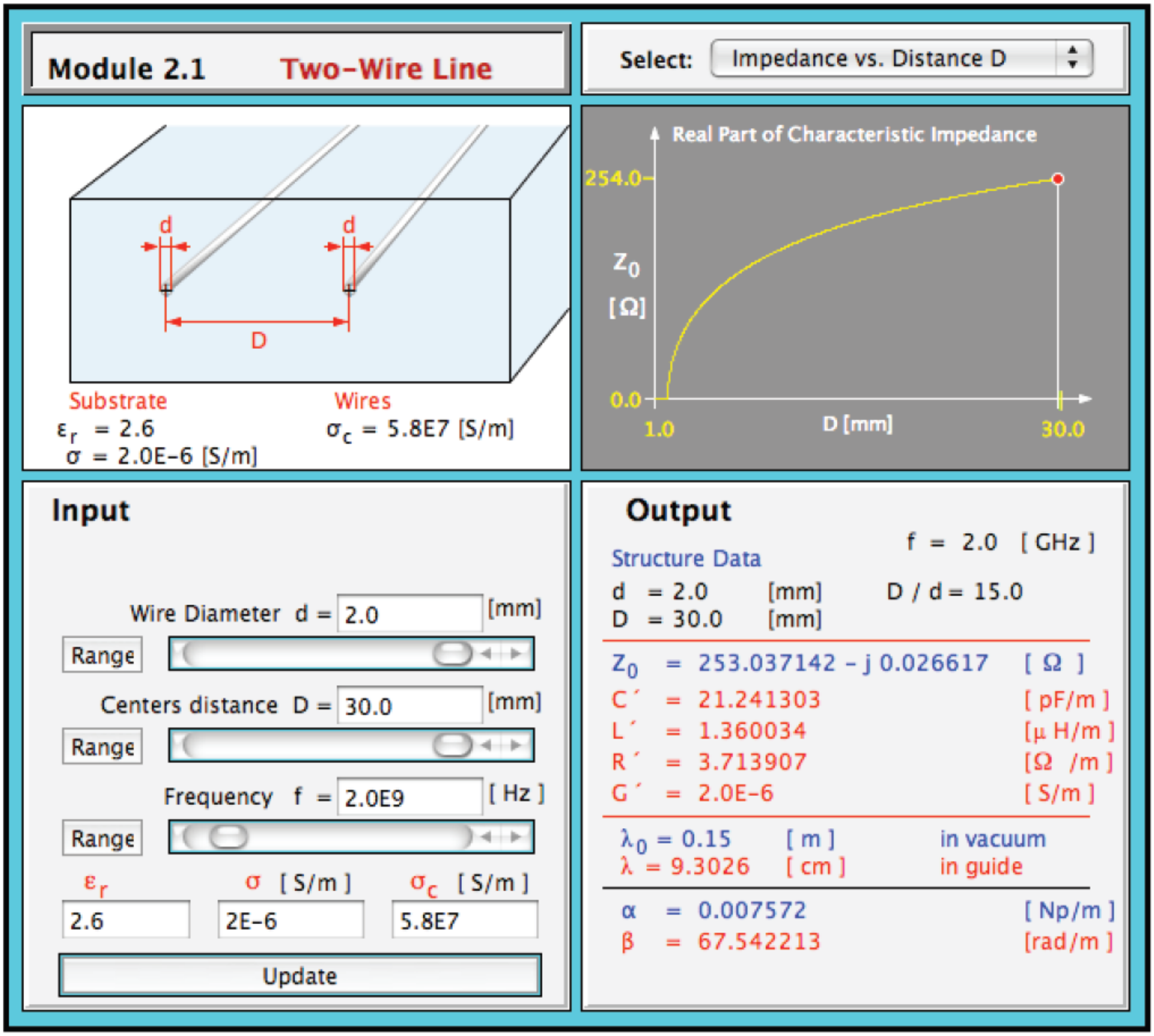
(a) Given:

$$
\begin{aligned}
f &= 2 \times 10^9 \text{ Hz}, \\
d &= 2 \times 10^{-3} \text{ m}, \\
D &= 3 \times 10^{-2} \text{ m}, \\
\sigma_c &= 5.8 \times 10^7 \text{ S/m (copper)}, \\
\varepsilon_r &= 2.6, \\
\sigma &= 2 \times 10^{-6} \text{ S/m}, \\
\mu &= \mu_c = \mu_0.
\end{aligned}
$$

From Table 2-1:

$$
\begin{aligned}
R_s &= \sqrt{\pi f \mu_c / \sigma_c} \\
&= [\pi \times 2 \times 10^9 \times 4\pi \times 10^{-7} / 5.8 \times 10^7]^{1/2} \\
&= 1.17 \times 10^{-2}\ \Omega, \\
R' &= \frac{2R_s}{\pi d} = \frac{2 \times 1.17 \times 10^{-2}}{2\pi \times 10^{-3}} = 3.71\ \Omega/\text{m}, \\
L' &= \frac{\mu}{\pi} \ln\left[(D/d) + \sqrt{(D/d)^2 - 1}\right] \\
&= 1.36 \times 10^{-6} \text{ H/m}, \\
G' &= \frac{\pi\sigma}{\ln[(D/d) + \sqrt{(D/d)^2 - 1}]} \\
&= 1.85 \times 10^{-6} \text{ S/m}, \\
C' &= \frac{G'\varepsilon}{\sigma} \\
&= \frac{1.85 \times 10^{-6} \times 8.85 \times 10^{-12} \times 2.6}{2 \times 10^{-6}} \\
&= 2.13 \times 10^{-11} \text{ F/m}.
\end{aligned}
$$

(b) Solution via Module 2.1:


Module 2.1
Two-Wire Line
Select:
Impedance vs. Distance D
d
d
D
Substrate
εr = 2.6
σ = 2.0E-6 [S/m]
Wires
σc = 5.8E7 [S/m]
Real Part of Characteristic Impedance
254.0
Z0
[Ω]
0.0
1.0
D [mm]
30.0
Input
Wire Diameter d = 2.0 [mm]
Range
Centers distance D = 30.0 [mm]
Range
Frequency f = 2.0E9 [Hz]
Range
εr
σ [S/m]
σc [S/m]
2.6
2E-6
5.8E7
Update
Output
f = 2.0 [GHz]
Structure Data
d = 2.0 [mm]
D / d = 15.0
D = 30.0 [mm]
Z0 = 253.037142 - j 0.026617 [Ω]
C´ = 21.241303 [pF/m]
L´ = 1.360034 [μ H/m]
R´ = 3.713907 [Ω /m]
G´ = 2.0E-6 [S/m]
λ0 = 0.15 [m] in vacuum
λ = 9.3026 [cm] in guide
α = 0.007572 [Np/m]
β = 67.542213 [rad/m]

Problem 2.3 Show that the transmission line model shown in Fig. P2.3 yields the same telegrapher's equations given by Eqs. (2.14) and (2.16).

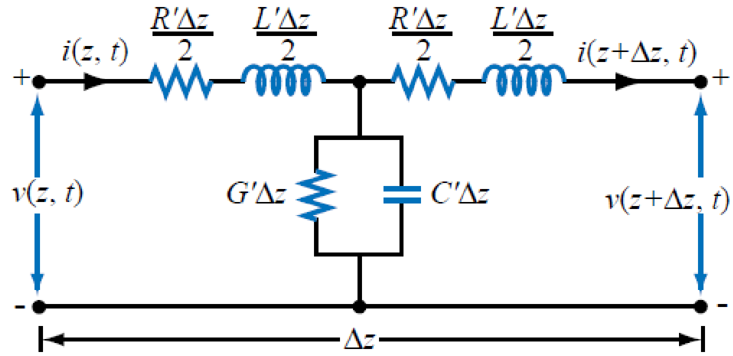


Figure P2.3: Transmission line model.

Solution: The voltage at the central upper node is the same whether it is calculated from the left port or the right port:

$$
\begin{aligned}
v(z+\tfrac{1}{2}\Delta z,t) &= v(z,t) - \tfrac{1}{2}R'\Delta z\, i(z,t) - \tfrac{1}{2}L'\Delta z\frac{\partial}{\partial t}i(z,t) \\
&= v(z+\Delta z,t) + \tfrac{1}{2}R'\Delta z\, i(z+\Delta z,t) + \tfrac{1}{2}L'\Delta z\frac{\partial}{\partial t}i(z+\Delta z,t).
\end{aligned}
$$

Recognizing that the current through the $G' \parallel C'$ branch is $i(z,t) - i(z+\Delta z,t)$ (from Kirchhoff's current law), we can conclude that

$$
i(z,t) - i(z+\Delta z,t) = G'\Delta z\, v(z+\tfrac{1}{2}\Delta z,t) + C'\Delta z\frac{\partial}{\partial t}v(z+\tfrac{1}{2}\Delta z,t).
$$

From both of these equations, the proof is completed by following the steps outlined in the text, ie. rearranging terms, dividing by Δz, and taking the limit as $\Delta z \to 0$.

Problem 2.4 A 1-GHz parallel-plate transmission line consists of 1.2-cm-wide copper strips separated by a 0.15-cm-thick layer of polystyrene. Appendix B gives $\mu_c = \mu_0 = 4\pi \times 10^{-7}$ (H/m) and $\sigma_c = 5.8 \times 10^7$ (S/m) for copper, and $\varepsilon_r = 2.6$ for polystyrene. Use Table 2-1 to determine the line parameters of the transmission line. Assume $\mu = \mu_0$ and $\sigma \simeq 0$ for polystyrene.

Solution:

$$R' = \frac{2R_s}{w} = \frac{2}{w}\sqrt{\frac{\pi f \mu_c}{\sigma_c}} = \frac{2}{1.2 \times 10^{-2}} \left(\frac{\pi \times 10^9 \times 4\pi \times 10^{-7}}{5.8 \times 10^7}\right)^{1/2} = 1.38 \quad (\Omega/\text{m}),$$

$$L' = \frac{\mu d}{w} = \frac{4\pi \times 10^{-7} \times 1.5 \times 10^{-3}}{1.2 \times 10^{-2}} = 1.57 \times 10^{-7} \quad (\text{H/m}),$$

$$G' = 0 \qquad \text{because } \sigma = 0,$$

$$C' = \frac{\varepsilon w}{d} = \varepsilon_0 \varepsilon_r \frac{w}{d} = \frac{10^{-9}}{36\pi} \times 2.6 \times \frac{1.2 \times 10^{-2}}{1.5 \times 10^{-3}} = 1.84 \times 10^{-10} \quad (\text{F/m}).$$

Problem 2.5 For a parallel-plate transmission line, the line parameters are given by:

$$R' = 1 \quad (\Omega/\text{m}),$$
$$L' = 167 \quad (\text{nH/m}),$$
$$G' = 0,$$
$$C' = 172 \quad (\text{pF/m}).$$

Find α, β, u_p, and Z_0 at 1 GHz.

Solution: At 1 GHz, $\omega = 2\pi f = 2\pi \times 10^9$ rad/s. Application of (2.22) gives:

$$\begin{aligned}
\gamma &= \sqrt{(R' + j\omega L')(G' + j\omega C')} \\
&= [(1 + j2\pi \times 10^9 \times 167 \times 10^{-9})(0 + j2\pi \times 10^9 \times 172 \times 10^{-12})]^{1/2} \\
&= [(1 + j1049)(j1.1)]^{1/2} \\
&= \left[\sqrt{1 + (1049)^2}\, e^{j\tan^{-1} 1049} \times 1.1 e^{j90^\circ}\right]^{1/2}, \qquad (j = e^{j90^\circ}) \\
&= \left[1049 e^{j89.95^\circ} \times 1.1 e^{j90^\circ}\right]^{1/2} \\
&= \left[1154 e^{j179.95^\circ}\right]^{1/2} \\
&= 34 e^{j89.97^\circ} = 34 \cos 89.97^\circ + j34 \sin 89.97^\circ = 0.016 + j34.
\end{aligned}$$

Hence,

$$\alpha = 0.016 \text{ Np/m},$$
$$\beta = 34 \text{ rad/m}.$$

$$u_p = \frac{\omega}{\beta} = \frac{2\pi f}{\beta} = \frac{2\pi \times 10^9}{34} = 1.85 \times 10^8 \text{ m/s}.$$

$$\begin{aligned}
Z_0 &= \left[\frac{R' + j\omega L'}{G' + j\omega C'}\right]^{1/2} \\
&= \left[\frac{1049 e^{j89.95^\circ}}{1.1 e^{j90^\circ}}\right]^{1/2} \\
&= \left[954 e^{-j0.05^\circ}\right]^{1/2} \\
&= 31 e^{-j0.025^\circ} \simeq (31 - j0.01)\ \Omega.
\end{aligned}$$

Problem 2.6 A coaxial line with inner and outer conductor diameters of 0.5 cm and 1 cm, respectively, is filled with an insulating material with $\varepsilon_r = 4.5$ and $\sigma = 10^{-3}$ S/m. The conductors are made of copper.

(a) Calculate the line parameters at 1 GHz.

(b) Compare your results with those based on CD Module 2.2. Include a printout of the screen display.

Solution: **(a)** Given

$$a = (0.5/2) \text{ cm} = 0.25 \times 10^{-2} \text{ m},$$
$$b = (1.0/2) \text{ cm} = 0.50 \times 10^{-2} \text{ m},$$

combining Eqs. (2.5) and (2.6) gives

$$\begin{aligned} R' &= \frac{1}{2\pi}\sqrt{\frac{\pi f \mu_c}{\sigma_c}}\left(\frac{1}{a}+\frac{1}{b}\right) \\ &= \frac{1}{2\pi}\sqrt{\frac{\pi(10^9 \text{ Hz})(4\pi\times 10^{-7} \text{ H/m})}{5.8\times 10^7 \text{ S/m}}}\left(\frac{1}{0.25\times 10^{-2} \text{ m}}+\frac{1}{0.50\times 10^{-2} \text{ m}}\right) \\ &= 0.788\ \Omega/\text{m}. \end{aligned}$$

From Eq. (2.7),

$$L' = \frac{\mu}{2\pi}\ln\left(\frac{b}{a}\right) = \frac{4\pi\times 10^{-7} \text{ H/m}}{2\pi}\ln 2 = 139 \text{ nH/m}.$$

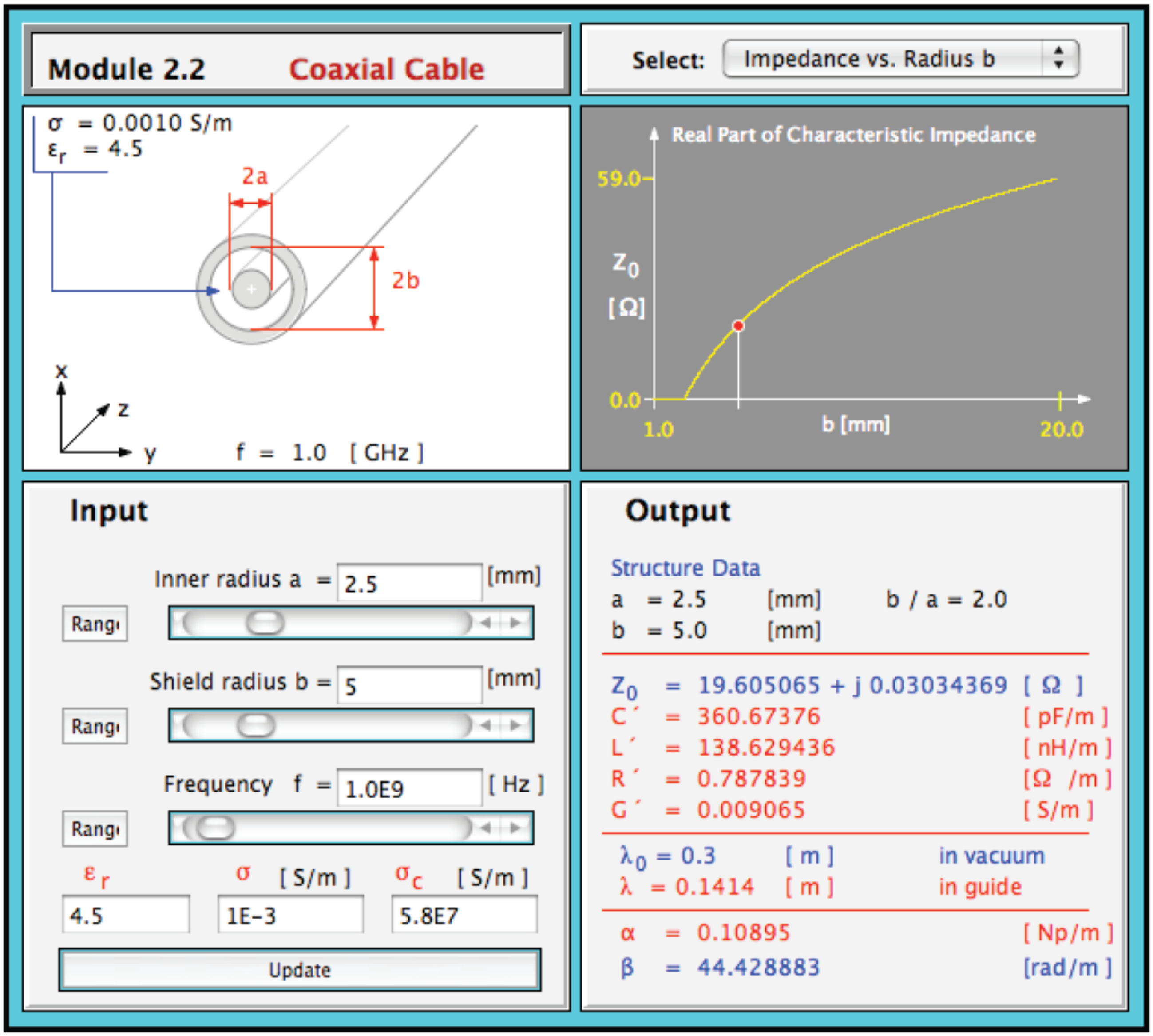
From Eq. (2.8),

$$G' = \frac{2\pi\sigma}{\ln(b/a)} = \frac{2\pi\times 10^{-3} \text{ S/m}}{\ln 2} = 9.1 \text{ mS/m}.$$

From Eq. (2.9),

$$C' = \frac{2\pi\varepsilon}{\ln(b/a)} = \frac{2\pi\varepsilon_r\varepsilon_0}{\ln(b/a)} = \frac{2\pi\times 4.5\times(8.854\times 10^{-12} \text{ F/m})}{\ln 2} = 362 \text{ pF/m}.$$

(b) Solution via Module 2.2:


Module 2.2
Coaxial Cable
Select:
Impedance vs. Radius b
σ = 0.0010 S/m
εr = 4.5
2a
2b
x
z
y
f = 1.0 [GHz]
Real Part of Characteristic Impedance
59.0
Z0
[Ω]
0.0
1.0
b [mm]
20.0
Input
Inner radius a = 2.5 [mm]
Shield radius b = 5 [mm]
Frequency f = 1.0E9 [Hz]
εr
σ [S/m]
σc [S/m]
4.5
1E-3
5.8E7
Update
Output
Structure Data
a = 2.5 [mm]
b / a = 2.0
b = 5.0 [mm]
Z0 = 19.605065 + j 0.03034369 [Ω]
C´ = 360.67376 [pF/m]
L´ = 138.629436 [nH/m]
R´ = 0.787839 [Ω /m]
G´ = 0.009065 [S/m]
λ0 = 0.3 [m] in vacuum
λ = 0.1414 [m] in guide
α = 0.10895 [Np/m]
β = 44.428883 [rad/m]

Problem 2.7 Find α, β, u_p, and Z_0 for the two-wire line of Problem 2.2. Compare results with those based on CD Module 2.1. Include a printout of the screen display.

Solution: From Problem 2.2:

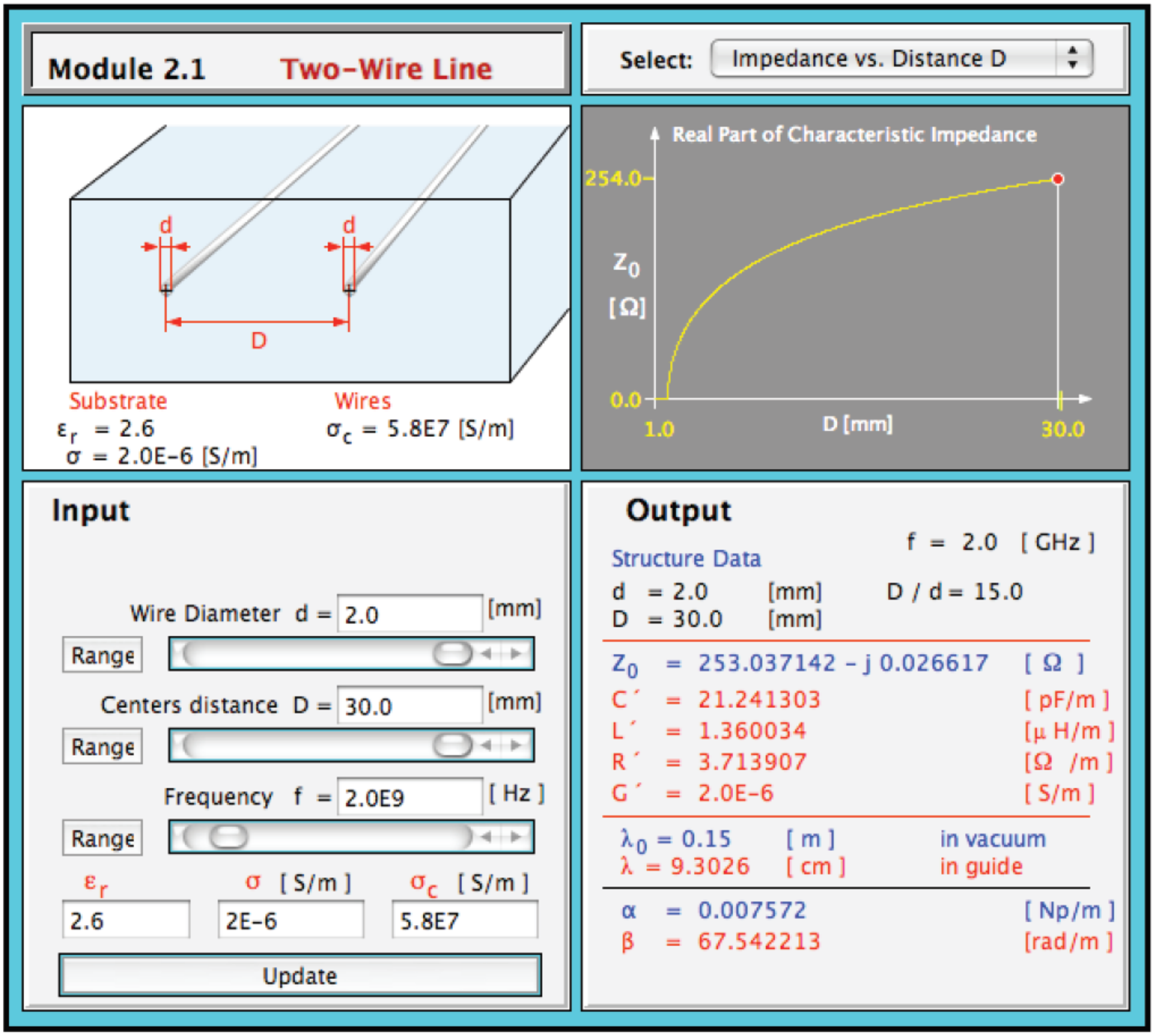
$$\begin{aligned}
R' &= 3.71\ \Omega/\text{m}, \\
L' &= 1.36\times10^{-6}\ \text{H/m}, \\
G' &= 1.85\times10^{-6}\ \text{S/m}, \\
C' &= 2.13\times10^{-11}\ \text{F/m}.
\end{aligned}$$

At 2 GHz:

$$\begin{aligned}
\gamma &= \sqrt{(R'+j\omega L')(G'+j\omega C')} \\
&= 0.0076 + j67.54.
\end{aligned}$$

Hence

$$\begin{aligned}
\alpha &= 0.0076\ \text{Np/m}, \\
\beta &= 67.54\ \text{rad/m}. \\
u_p &= \frac{\omega}{\beta} = \frac{2\pi\times2\times10^9}{67.54} = 1.86\times10^8\ \text{m/s}, \\
Z_0 &= \sqrt{\frac{R'+j\omega L'}{G'+j\omega C'}} = 253\ \Omega.
\end{aligned}$$


Module 2.1
Two-Wire Line
Select:
Impedance vs. Distance D
d
d
D
Substrate
εr = 2.6
σ = 2.0E-6 [S/m]
Wires
σc = 5.8E7 [S/m]
Real Part of Characteristic Impedance
254.0
Z0
[Ω]
0.0
1.0
D [mm]
30.0
Input
Wire Diameter d = 2.0 [mm]
Range
Centers distance D = 30.0 [mm]
Range
Frequency f = 2.0E9 [Hz]
Range
εr
σ [S/m]
σc [S/m]
2.6
2E-6
5.8E7
Update
Output
f = 2.0 [GHz]
Structure Data
d = 2.0 [mm]
D / d = 15.0
D = 30.0 [mm]
Z0 = 253.037142 - j 0.026617 [Ω]
C´ = 21.241303 [pF/m]
L´ = 1.360034 [μ H/m]
R´ = 3.713907 [Ω /m]
G´ = 2.0E-6 [S/m]
λ0 = 0.15 [m] in vacuum
λ = 9.3026 [cm] in guide
α = 0.007572 [Np/m]
β = 67.542213 [rad/m]

Problem 2.8 Find α, β, u_{p}, and Z_0 for the coaxial line of Problem 2.6. Verify your results by applying CD Module 2.2. Include a printout of the screen display.

Solution: From Eq. (2.22),

$$\begin{aligned}
\gamma &= \sqrt{(R' + j\omega L')(G' + j\omega C')} \\
&= \sqrt{(0.788\ \Omega/\mathrm{m}) + j(2\pi \times 10^9\ \mathrm{s}^{-1})(139 \times 10^{-9}\ \mathrm{H/m})} \\
&\quad \times \sqrt{(9.1 \times 10^{-3}\ \mathrm{S/m}) + j(2\pi \times 10^9\ \mathrm{s}^{-1})(362 \times 10^{-12}\ \mathrm{F/m})} \\
&= (109 \times 10^{-3} + j44.5)\ \mathrm{m}^{-1}.
\end{aligned}$$

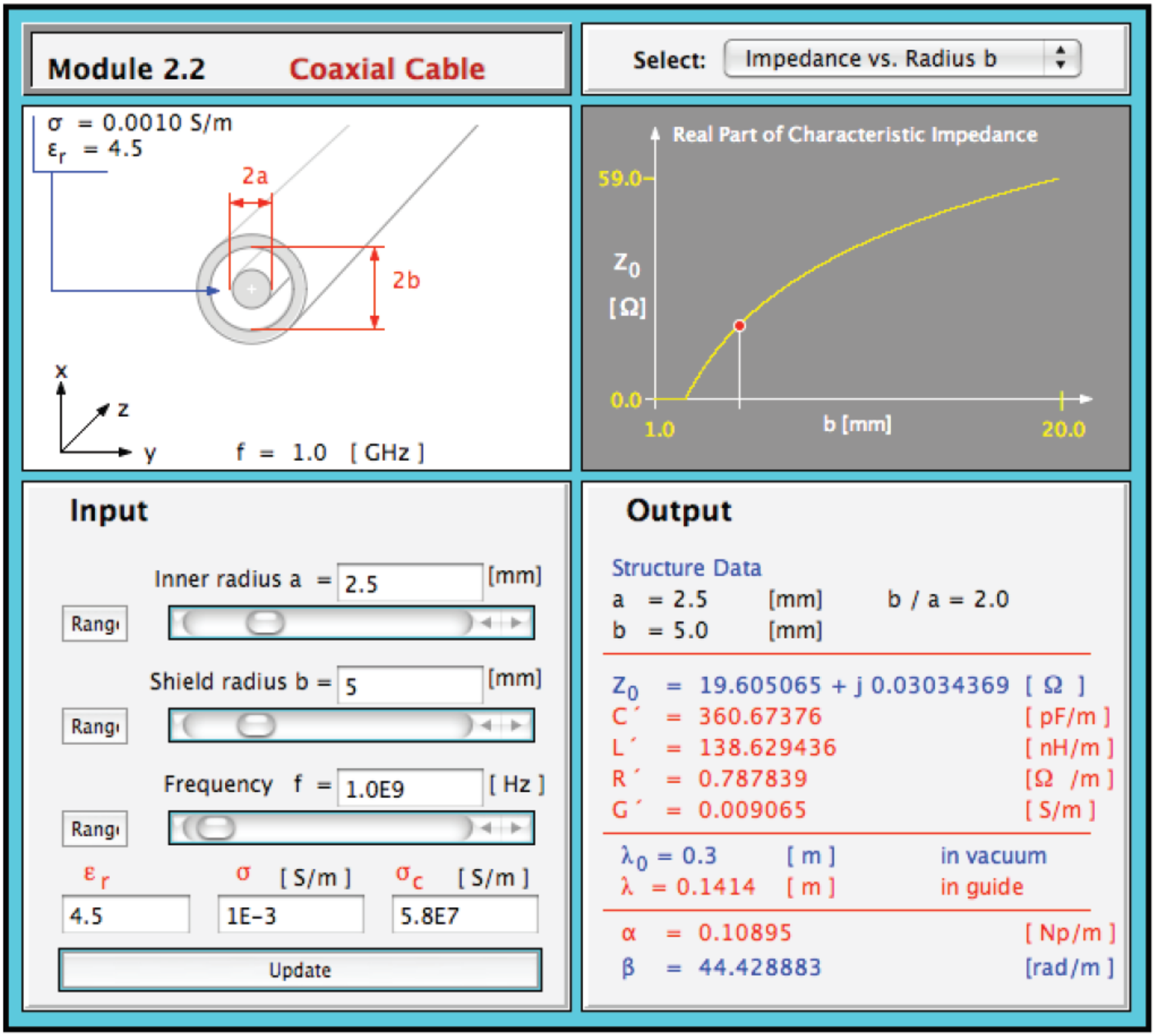
Thus, from Eqs. (2.25a) and (2.25b), $\alpha = 0.109$ Np/m and $\beta = 44.5$ rad/m.

From Eq. (2.29),

$$\begin{aligned}
Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} &= \sqrt{\frac{(0.788\ \Omega/\mathrm{m}) + j(2\pi \times 10^9\ \mathrm{s}^{-1})(139 \times 10^{-9}\ \mathrm{H/m})}{(9.1 \times 10^{-3}\ \mathrm{S/m}) + j(2\pi \times 10^9\ \mathrm{s}^{-1})(362 \times 10^{-12}\ \mathrm{F/m})}} \\
&= (19.6 + j0.030)\ \Omega.
\end{aligned}$$

From Eq. (2.33),

$$u_{\mathrm{p}} = \frac{\omega}{\beta} = \frac{2\pi \times 10^9}{44.5} = 1.41 \times 10^8\ \mathrm{m/s}.$$


Module 2.2
Coaxial Cable
Select:
Impedance vs. Radius b
σ = 0.0010 S/m
ε r = 4.5
2a
2b
x
z
y
f = 1.0 [GHz]
Real Part of Characteristic Impedance
59.0
Z 0
[Ω]
0.0
1.0
b [mm]
20.0
Input
Inner radius a =
2.5
[mm]
Shield radius b =
5
[mm]
Frequency f =
1.0E9
[Hz]
ε r
σ [S/m]
σ c [S/m]
4.5
1E-3
5.8E7
Update
Output
Structure Data
a = 2.5 [mm]
b / a = 2.0
b = 5.0 [mm]
Z 0 = 19.605065 + j 0.03034369 [Ω]
C ′ = 360.67376 [pF/m]
L ′ = 138.629436 [nH/m]
R ′ = 0.787839 [Ω /m]
G ′ = 0.009065 [S/m]
λ 0 = 0.3 [m] in vacuum
λ = 0.1414 [m] in guide
α = 0.10895 [Np/m]
β = 44.428883 [rad/m]